

Decoherence according to Environment and Self Induced Decoherences.

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A generalized decoherence formalism that can be used both in open (using Environment Induced Decoherence-EID) and closed (using Self Induced Decoherence-SID) quantum systems is sketched.

I. INTRODUCTION

Quantum mechanics is essentially characterized by the superposition principle and the phenomenon of interference. Therefore, any attempt to explain how classicality emerges from quantum behavior must include two elements: a process through which interference vanishes, and a superselection rule that precludes superpositions. Decoherence is the process that eliminates interference and leads to the rule that selects the candidates for classical states.

Schematically, three periods can be identified in the development of the theory:

Closed systems period (van Kampen, van Hove, Daneri *et al.* [1]). In order to understand how classical macroscopic features arise from quantum microscopic behavior, "gross" observables are defined. The states that are indistinguishable for a macroscopic observer are described by the same coarse-grained state $\rho_G(t)$. When the evolution of $\rho_G(t)$ (or of the expectation value of the gross observables) is studied, it is proved that $\rho_G(t)$ reaches equilibrium in a relaxation time t_R ; therefore, $\rho_G(t)$ decoheres in its own eigenbasis after a decoherence time $t_D = t_R$. The main problem of this period was the fact that t_R turned out to be too long to account for experimental data (see [2]).

Open systems period. An open system S is considered in interaction with its environment E , and the evolution of the reduced state $\rho_S(t) = \text{Tr}_E \rho(t)$ is studied. The so called "environment induced decoherence" (EID) (Zeh and Zurek [3]) proves that, since the interference terms of $\rho_S(t)$ rapidly vanish, $\rho_S(t)$ decoheres in an adequate pointer basis after a short decoherence time $t_D = t_{DS}$. This result solved the main problem of the first period.

Closed and open systems period. Although at present EID is still considered the "orthodoxy" in the subject ([2]), other approaches have been proposed to face the problems of EID, in particular, the closed-system problem (Diosi, Milburn, Penrose, Casati and Chirikov, Adler [4]). Some of these methods are clearly "non-dissipative" (Ford and O'Connell [5]), that is, not based on the dissipation of energy from the system to the environment. Among them, we have developed the self-induced decoherence (SID) approach, according to which a closed quantum system with continuous spectrum may decohere by destructive interference, and reaches a final state where the classical limit can be obtained ([6]).

In spite of the fact that the theories of decoherence in closed and open systems coexist in the third period, in the literature both kinds of approaches are usually conceived as antagonistic scenarios for decoherence or even as formalism dealing with different physical phenomena ([7]). In this paper we will argue that this is not the case; on the contrary, the two kinds of theories can be subsumed under a more general theoretical framework that shows their complementary character.

II. OBSERVABLES, MEAN VALUES, AND WEAK LIMITS

As emphasized by Omnés ([2]), decoherence is a particular case of the phenomenon of irreversibility, which leads to the following problem. Since the quantum state $\rho(t)$ evolves unitarily, it cannot reach a final equilibrium state for $t \rightarrow \infty$. Therefore, if the non-unitary evolution towards equilibrium is to be accounted for, a further element has to be added, which consists in the splitting of the maximal information about the system into a relevant part and an irrelevant part: whereas the irrelevant part is discarded, the relevant part reaches a final equilibrium situation. In some cases, the irrelevant part can be conceived as an environment, but this is not always necessary. When this idea is rephrased in operator language, the phenomenon of decoherence can be explained in three general steps:

- 1.- The set \mathcal{O} of relevant observables is defined. The observables $O_R \in \mathcal{O}$ restrict the maximal information given by the set of all the possible observables of the system.
- 2.- The expectation value $\langle O_R \rangle_{\rho(t)}$ is computed, for any $O_R \in \mathcal{O}$.
- 3.- It is proved that $\langle O_R \rangle_{\rho(t)}$ reaches a final equilibrium value expressed by the limit of $\langle O_R \rangle_{\rho(t)}$ or the weak limit of $\rho(t)$:

$$\lim_{t \rightarrow \infty} \langle O_R \rangle_{\rho(t)} = \langle O_R \rangle_{\rho_*} \quad \text{or} \quad W \lim_{t \rightarrow \infty} \rho(t) = \rho_*$$

These three stages can be found in both open and closed systems; in particular, we will find them in the example of section IV.

When decoherence in closed and open systems is understood in the context of this common general framework, the relationship between both cases can be studied. Let us consider a large closed system U ("the universe") that can be split into many subsystems S_i such that $U = \bigcup_i S_i$, each one of them with its own environment $E_i = U \setminus S_i$. Let $\rho(t)$ be the state of U and $\rho_{S_i}(t)$ the state of S_i . Under very general conditions, it can be proved that:

(i) The decoherence of the closed system U yields the decoherence of all its open subsystems S_i in interaction with their environments E_i . In other words, if $\rho(t)$ reaches an equilibrium state ρ_* (according to SID), which is diagonal in its pointer basis, then all the $\rho_{S_i}(t)$ also reach equilibrium states ρ_{S_i*} (according to EID), each one diagonal in its own pointer basis.

(ii) The decoherence of all the open subsystems S_i in interaction with their environments E_i yields the decoherence of the closed system U . In other words, if all the $\rho_{S_i}(t)$ reach equilibrium states ρ_{S_i*} (according to EID), each one diagonal in its pointer basis, then $\rho(t)$ reaches an equilibrium state ρ_* (according to SID), which is diagonal in its own pointer basis.

Of course, each process has its own decoherence time: these times may be different and, eventually, some of them may be infinite (in this case, decoherence is merely theoretical). We will develop this general idea in mathematical terms elsewhere. Nevertheless, this shows that the formalisms for open and closed systems are complementary, and both cooperate in the understanding of the same physical reality.

III. DECOHERENCE TIME, TRIVIALITY, AND COMPLEXITY

For open systems, the decoherence time t_{DS} obtained in several models studied by EID (precisely, the characteristic time of the diagonalization of $\rho_S(t)$ in a pointer basis obtained by means of the predictability sieve criterion) is given by eq.(47) of [9] or by eq.(3.136) of [10].

For closed systems, in paper [11] we have found the decoherence time t_{DU} by the SID method, and we have explained that such a time is *infinite* when the Hamiltonian and the initial conditions are *trivial*, (i.e., just with real poles in the analytical continuation of the resolvent and the initial conditions). In the Appendix B of that paper we, have showed how the method can describe a two-times evolution (that can be easily generalized to an n-times evolution). Let us rephrase that appendix: by choosing $V^{(1)}$ as representing the interaction between a proper system S and its environment E , $V^{(2)}$ as representing the interaction of the parts of the environment E among themselves, and $V^{(1)}(\omega, \omega') \gg V^{(2)}(\omega, \omega')$, we will find that the first interaction gives the decoherence time t_{DS} of the proper system S and the second interaction gives the decoherence time t_{DU} of the whole system U . These interactions are what preclude triviality and introduce complexity. Moreover, as $V^{(1)}(\omega, \omega') \gg V^{(2)}(\omega, \omega')$ and $t_D \sim \frac{\hbar}{V}$, we obtain $t_{DS} \ll t_{DU}$. As an example (see [11] for details), if we assume a microscopic self-environment interactions $V^{(2)} \approx 1 \text{ eV}$, and a macroscopic oscillator-environment interaction $V^{(1)} \approx 10^{23} \text{ eV}$, we have $t_{DU} \approx 10^{-15} \text{ s}$ and $t_{DS} \approx 10^{-37} \text{ s} - 10^{-39} \text{ s}$. As a consequence, we certainly obtain $t_{DS} \ll t_{DU}$.

IV. A WELL KNOWN MODEL

Let us consider a spin system S_0 (with Hilbert space \mathcal{H}_0) and a set of N spin systems S_i (with Hilbert spaces \mathcal{H}_i) coupled by the Hamiltonian (see [7])

$$H = H_{SE} = \frac{1}{2}(|0\rangle\langle 0| - |1\rangle\langle 1|) \sum_{i=0}^N g_i (|\uparrow_i\rangle\langle \uparrow_i| - |\downarrow_i\rangle\langle \downarrow_i|) \bigotimes_{j \neq i}^N I_j \quad (1)$$

We will also suppose that the free Hamiltonians are $H_S = H_E = 0$.

Let us consider a pure state $|\psi_0\rangle = (a|0\rangle + b|1\rangle) \bigotimes_i^N (\alpha_i |\uparrow_i\rangle + \beta_i |\downarrow_i\rangle)$, where α_i and β_i are aleatory coefficients such that $|\alpha_i|^2 + |\beta_i|^2 = 1$. The state $|\psi_0\rangle$ evolves as $|\psi(t)\rangle = a|0\rangle|\mathcal{E}_0(t)\rangle + b|1\rangle|\mathcal{E}_1(t)\rangle$, where $|\mathcal{E}_0(t)\rangle = |\mathcal{E}_1(-t)\rangle = \bigotimes_i^N (\alpha_i e^{ig_i t/2} |\uparrow_i\rangle + \beta_i e^{-ig_i t/2} |\downarrow_i\rangle)$, and its matrix version is $\rho(t) = |\psi(t)\rangle\langle \psi(t)|$. Now we can develop the three steps introduced in Section II in the case of this model.

Step 1: We will consider the observables $O \in \mathcal{O}$ such that

$$O = (s_{00}|0\rangle\langle 0| + s_{01}|0\rangle\langle 1| + s_{10}|1\rangle\langle 0| + s_{11}|1\rangle\langle 1|) \bigotimes_{i=1}^N (\epsilon_{\uparrow\uparrow}^{(i)} |\uparrow_i\rangle\langle\uparrow_i| + \epsilon_{\downarrow\downarrow}^{(i)} |\downarrow_i\rangle\langle\downarrow_i| + \epsilon_{\downarrow\uparrow}^{(i)} |\downarrow_i\rangle\langle\uparrow_i| + \epsilon_{\uparrow\downarrow}^{(i)} |\uparrow_i\rangle\langle\downarrow_i|)$$

where $s_{00}, s_{11}, \epsilon_{\uparrow\uparrow}^{(i)}, \epsilon_{\downarrow\downarrow}^{(i)}$ are real numbers and $s_{01} = s_{10}^*$, $\epsilon_{\uparrow\downarrow}^{(i)} = \epsilon_{\downarrow\uparrow}^{(i)*}$ are complex numbers. These observables are not completely general, but they are the relevant observables for this case (see [7] eq.(18)).

Stage 2: The expectation value of O in the state $|\psi(t)\rangle$ reads

$$\langle O \rangle_{\psi(t)} = (|a|^2 s_{00} + |b|^2 s_{11}) \Gamma_0(t) + 2 \operatorname{Re}[ab^* s_{10} \Gamma_1(t)]$$

where

$$\Gamma_0(t) = \prod_{i=1}^N [|\alpha_i|^2 \epsilon_{\uparrow\uparrow}^{(i)} + |\beta_i|^2 \epsilon_{\downarrow\downarrow}^{(i)} + \alpha_i^* \beta_i \epsilon_{\uparrow\downarrow}^{(i)} e^{-ig_i t} + (\alpha_i^* \beta_i \epsilon_{\uparrow\downarrow}^{(i)})^* e^{ig_i t}] \quad (2)$$

$$\Gamma_1(t) = \prod_{i=1}^N [|\alpha_i|^2 \epsilon_{\uparrow\uparrow}^{(i)} e^{ig_i t} + |\beta_i|^2 \epsilon_{\downarrow\downarrow}^{(i)} e^{-ig_i t} + \alpha_i^* \beta_i \epsilon_{\uparrow\downarrow}^{(i)} + (\alpha_i^* \beta_i \epsilon_{\uparrow\downarrow}^{(i)})^*] \quad (3)$$

Let us consider two special cases of observables:

(a) If $\epsilon_{\uparrow\uparrow}^{(i)} = \epsilon_{\downarrow\downarrow}^{(i)} = 1$, $\epsilon_{\uparrow\downarrow}^{(i)} = 0$, we are in the typical case of EID. In this case,

$$O_{S_0} = (s_{00}|0\rangle\langle 0| + s_{01}|0\rangle\langle 1| + s_{10}|1\rangle\langle 0| + s_{11}|1\rangle\langle 1|) \bigotimes_{i=0}^N I_i$$

The expectation value of these observables is given by

$$\langle O_S \rangle_{\psi(t)} = |\alpha_i|^2 s_{00} + |\beta_i|^2 s_{01} + \operatorname{Re}[ab^* s_{10} r(t)] \quad (4)$$

where $r(t) = \langle \mathcal{E}_1(t) | \mathcal{E}_0(t) \rangle$ and

$$|r(t)|^2 = \prod_{i=1}^N (|\alpha_i|^4 + |\beta_i|^4 + 2|\alpha_i|^2 |\beta_i|^2 \cos 2g_i t) \quad (5)$$

These are the observables that "observe" only the spin system S_0 .

(b) But we can also define the observables that "observe" just *one* spin system S_j of the environment:

$$O_{S_j} = I_{S_0} \otimes O_j \bigotimes_{i \neq j} I_{S_i}$$

with

$$O_j = \epsilon_{\uparrow\uparrow}^{(j)} |\uparrow_j\rangle\langle\uparrow_j| + \epsilon_{\downarrow\downarrow}^{(j)} |\downarrow_j\rangle\langle\downarrow_j| + \epsilon_{\downarrow\uparrow}^{(j)} |\downarrow_j\rangle\langle\uparrow_j| + \epsilon_{\uparrow\downarrow}^{(j)} |\uparrow_j\rangle\langle\downarrow_j|$$

where $\epsilon_{\uparrow\uparrow}^{(j)}, \epsilon_{\downarrow\downarrow}^{(j)}, \epsilon_{\uparrow\downarrow}^{(j)}$ are now generic. The expectation value of these observables is given by

$$\begin{aligned} \langle O_{S_j} \rangle_{\psi(t)} &= \langle \psi(t) | O_{S_j} | \psi(t) \rangle = |a|^2 (|\alpha_j|^2 \epsilon_{\uparrow\uparrow}^{(j)} + |\beta_j|^2 \epsilon_{\downarrow\downarrow}^{(j)} + \alpha_j \beta_j^* \epsilon_{\uparrow\downarrow}^{(j)} e^{-ig_j t} + \alpha_j^* \beta_j \epsilon_{\downarrow\uparrow}^{(j)} e^{ig_j t}) + \\ &|b|^2 (|\alpha_j|^2 \epsilon_{\uparrow\uparrow}^{(j)} + |\beta_j|^2 \epsilon_{\downarrow\downarrow}^{(j)} + \alpha_j \beta_j^* \epsilon_{\uparrow\downarrow}^{(j)} e^{ig_j t} + \alpha_j^* \beta_j \epsilon_{\downarrow\uparrow}^{(j)} e^{-ig_j t}) \end{aligned} \quad (6)$$

Step 3: Let us now compute the time evolution in the two cases.

(a) Since $\max_t (|\alpha_i|^4 + |\beta_i|^4 + 2|\alpha_i|^2 |\beta_i|^2 \cos 2g_i t) = 1$ and $\min_t (|\alpha_i|^4 + |\beta_i|^4 + 2|\alpha_i|^2 |\beta_i|^2 \cos 2g_i t) = (2|\alpha_i|^2 - 1)^2$, the aleatory numbers $(|\alpha_i|^4 + |\beta_i|^4 + 2|\alpha_i|^2 |\beta_i|^2 \cos 2g_i t)$ fluctuate between 1 and $(2|\alpha_i|^2 - 1)^2$. Then, from eq.(5) we obtain

$$\lim_{N \rightarrow \infty} r(t) = 0$$

and from eq.(4) we obtain the weak limit

$$\lim_{t \rightarrow \infty} \langle O_{S_0} \rangle_{\psi(t)} = |\alpha_i|^2 s_{00} + |\beta_i|^2 s_{11} = \langle O_{S_0} \rangle_{\rho_{S^*}}$$

where

$$\rho_{S^*} = \begin{pmatrix} |\alpha_i|^2 & 0 \\ 0 & |\beta_i|^2 \end{pmatrix}$$

This means that the proper system S_0 decoheres, according to EID, in a finite decoherence time t_{DS_0} (in fact, see $r(t)$ for $N = 20$ and $N = 100$ in fig.1 of [7]).

(b) But let us consider now the evolution of the spin system S_j given by eq.(6): $\langle O_{S_j} \rangle_{\psi(t)}$ just oscillates and, as a consequence, it has no limit. Therefore, the generic spin system S_j certainly does not decohere. This is not surprising if we recall that, in the Hamiltonian (1), the spin systems of the environment $E = \bigcup_i S_i$ are uncoupled: each spin system freely evolves and, for this reason, the environment is unable to reach a final stable state.¹ We could also have obtained this result by means of SID since, as explained in Section III, if $V_{(2)} = 0$, then $t_{DU} \rightarrow \infty$.

This model shows that, by using the observables of points (a) and (b), we gain a complete knowledge of the behavior of the system. The Hamiltonian (1) is not symmetric with respect to S_0 and S_j , since S_0 is coupled to all the S_j , but the S_j are not coupled to each other but only coupled to S_0 . In other words, since S_0 is not trivially coupled, it decoheres in a finite time t_{DS_0} (see fig.1 of [7]), whereas the S_j are uncoupled or "trivially coupled" and, therefore, they do not decohere.

But the point to emphasize here is that we could have deduced this result directly from Section III: the environment $E = \bigcup_i S_i$ does not decohere (namely, it has an infinite decoherence time). As a consequence, the whole system $U = S_0 \cup E$ does not decohere and it would be a miracle that it did (see figs. 2, 3 and 4 of [7]). Our general framework allows us to understand the well known model of [7] and [9] from a broader perspective: it can be proved that $t_{DS_0} \ll t_{DU} = \infty$,² and the results of EID and SID can be shown to agree.

V. CONCLUSION.

The formalism sketched in this paper encompasses both EID and SID (and probably other decoherence approaches). It is supported by the experimental basis obtained both in EID and SID. EID has a large amount of experimental confirmations (see [10]). The best result of SID is the complete description of the classical limit of quantum systems (see [12]), and the fact that the classical mechanics so obtained is experimentally proved beyond all doubts. Moreover, according to [11], decoherence times computed by EID and by SID are of similar order.

The new formalism supplies a solution to the main problems of the EID approach:

- i.- Closed systems do decohere and their decoherence times can be computed.
- ii.- The "looming big" problem of EID, i.e. the fact that it does not provide a criterion to decide where to place the cut between "the" proper system S and "the" environment E in the closed system U , is now dissolved. In fact, the complete study of the system U requires several partitions $U = S_i \cup E_i$. The choice of these partitions is arbitrary, but the nature of each partition is given by the Hamiltonian. In fact, the example of section IV, with Hamiltonian (1), shows that there are only two kinds of spin systems to be studied: S_0 and a generic S_j . Only once the two subsystems are studied, but not before, the whole closed system U is completely understood.³

iii.- The problem of rigorously defining a dynamical pointer basis (valid for all times) via, e.g., the predictability sieve criterion, remains unsolved. But the final pointer basis, where decoherence appears in the weak limits, is completely well defined.

Finally, it is worth stressing that the study of the role of complexity in decoherence presented in paper [13], where quantum non-integrable systems are described, may solve problems like those of paper [14], where quantum chaos has obviously to be invoked.

¹They are only indirectly coupled through S_0 , but this interaction becomes negligible when $N \rightarrow \infty$.

²From this perspective, all the criticisms to SID of paper [7] disappear, since they were formulated from a wrong viewpoint. The model does not show that the destructive interference of the off-diagonal terms of $\rho(t)$ is not efficient in all cases, as it is claimed; it merely proves that the whole closed system does not decohere when the environment Hamiltonian is trivial, which is a universally accepted fact. Nevertheless, the criticisms of [7] are far from being trivial: we needed several months to understand the puzzle.

³This strategy was not followed in paper [7], where the system was not completely studied, and thus a wrong conclusion was obtained.

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